Nuclear Collective Tunneling Transitions
between Hartree States Deformed in Quadrupole Symmetry

M. Maruyama

Department of Physics, Tohoku University, Sendai 980-8578, Japan

T. Kohmura

Department of Business Administration, Josai University, Sakado 350-0295, Japan

Y. Hashimoto

Graduate School of Pure and Applied Sciences, University of Tsukuba, Tsukuba 305-8571, Japan

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In a nucleus which has two Hartree states deformed in quadrupole symmetry, i.e., prolate state and oblate state, the nuclear residual interaction derived in the present theory beyond the Hartree approximation acts as the restoring force for the spherical symmetry of the nuclear system to be recovered so that the deformed nucleus makes collective tunneling transitions between prolate states and oblate states. We derive the Hamiltonian that is effective not only for the Hartree states but also for the collective tunneling transitions between the Hartree states. Solving the secular equation for the effective Hamiltonian on the basis states expressed in terms of SU(5) algebra, we analyze the nuclear collective tunneling transitions between a prolate state and an oblate state in the real-time description. The collective tunneling transitions are discussed to proceed through tri-axially asymmetric deformed states.

I. INTRODUCTION

A nucleus is a self-contained system of strongly interacting nucleons, where the concept of Hartree is very useful to understand the mechanism of the system to be self-consistently bound: The Hartree approximation for a nucleus determines the nuclear shape and single-particle states. The nucleus of ^{28}Si, for example, has two, i.e., prolate and oblate, deformed Hartree states. A Hartree state of an open-shell nucleus is characterized by the deformed Hartree potential, which is composed of the meson mean fields proper to the Hartree state [1][2].

The present theory beyond the Hartree approximation takes into account the fluctuation of the meson mean field proper to other Hartree states to take place in one Hartree state. The meson mean-field fluctuation proper to other Hartree states yields a residual interaction, which gives rise to the collective tunneling transitions from the Hartree state to another Hartree state in order to restore the spherical symmetry. Taking advantage of this picture of nuclear collective tunneling, in the previous papers [1][2], we derived a Hamiltonian effective not only for deformed Hartree states but also for the collective tunneling transitions between the deformed Hartree states. The physical quantities in the Hamiltonian are determined by the Hartree calculations of the deformed states.

In this paper, we solve the secular equation for the nuclear effective Hamiltonian on the basis states expressed in terms of SU(5) algebra and analyze the collective tunneling transitions between the two deformed Hartree states in the real-time description in the present theory beyond the Hartree approximation. We discuss that the nuclear residual interaction, exciting d-state nucleons to s state and vice versa, acts as the restoring force for the spherical symmetry of the nuclear system to be recovered so that the deformed nucleus makes collective tunneling transitions between prolate states and oblate states. The nucleus ^{28}Si in the course of collective tunneling transitions between a prolate state and an oblate state proceeds through tri-axially asymmetric deformed states [3][4].

II. HAMILTONIAN EFFECTIVE FOR HARTREE STATES AND FOR COLLECTIVE TUNNELING

In the previous papers [1][2], we derived a Hamiltonian effective not only for two deformed Hartree states but also for the collective tunneling transitions between the two deformed Hartree states. The physical quantities in the effective Hamiltonian are determined by the Hartree calculations of the deformed states.

Here we briefly review the formulation for the collective tunneling transitions between Hartree states in the description based on the non-relativistic meson mean-field calculations for the Hartree states. In the present description, for simplicity's sake, we adopt one sort of meson field in place of σ- and ω-mesons and endorse the nuclear saturation by the truncation of the meson space. The non-relativistic nuclear field Hamiltonian is written as

$$H_{\text{nucl}} = \int [\psi^\dagger(\vec{r})(-\frac{\nabla^2}{2M})\psi(\vec{r}) + \frac{1}{2}\{\pi^2(\vec{r}) + \phi(\vec{r})(-\nabla^2 + m^2)\phi(\vec{r})\}$$
$$-g\phi(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})]d^3r, \tag{1}$$

where ψ is nucleon field, ϕ and π are meson fields, and M, m and g are nucleon mass, meson mass and meson-nucleon coupling constant, respectively.

We first solve the Hartree problems for a nuclear system which has two Hartree states Ψ_i and Ψ_f. The two Hartree states are specified by the meson mean fields φ^i and φ^f, respectively. In terms of the meson mean fields φ^i and φ^f in the two Hartree states determined in the Hartree calculations, we formulate the collective tunneling transitions between the two Hartree states. The meson mean field φ, which varies from φ^i to φ^f in the course of the collective tunneling, acts as a residual interaction to steer the nuclear collective tunneling transitions from Ψ_i to Ψ_f in the present theory beyond the Hartree approximation.

We derive a Hamiltonian effective for the Hartree states and also for the collective tunneling transitions between the Hartree states [1][2]. In terms of the meson mean fields φ^i and φ^f in the two Hartree states Ψ_i and Ψ_f, we define the tunneling steering field to be

$$\varphi_s(\vec{r}) = \frac{1}{2}\{\varphi^f(\vec{r}) - \varphi^i(\vec{r})\}, \tag{2}$$

and the principal mean field to be

$$\varphi_p(\vec{r}) = \frac{1}{2}\{\varphi^i(\vec{r}) + \varphi^f(\vec{r})\}. \tag{3}$$

Hartree states deformed in quadrupole symmetry, either prolate or oblate, are one of the features of the nuclei that have some valence nucleons in the s and d major shell. We formulate the collective tunneling transitions of s-d shell nuclei between a prolate Hartree state and an oblate Hartree state. A typical example of the nuclei that exhibit both an oblate and a prolate Hartree state with an almost degenerate energy is the nucleus of ^{28}Si [5, 6, 7], which has 6 protons and 6 neutrons in the s and d major shell. While a configuration of nucleons in the non-spherical d states deforms the Hartree potential, the nuclear residual interaction that is of spherical symmetry gives rise to collective tunneling transitions between the prolate Hartree state and the oblate Hartree state.

Suppose that the nucleus ^{28}Si is polarized along z-axis in the two, i.e., prolate and oblate, deformed Hartree states. The two Hartree states are symmetric to each other with respect to the deformation parameter [8], i.e., the principal mean field in Eq. (3) is spherically symmetric,

$$\varphi_p(\vec{r}) = \varphi_p(r)Y_{00}(\theta, \varphi), \tag{4}$$

and the tunneling steering field in Eq. (2) is of quadrupole symmetry,

$$\varphi_s(\vec{r}) = \varphi_s(r)Y_{20}(\theta, \varphi). \tag{5}$$

We quantize the meson field $\phi(\vec{r})$ in terms of only the three essential fields, i.e., the principal mean field $\varphi_p(\vec{r})$, the tunneling steering field of quadrupole symmetry,

$$\bar{\varphi}_{20}(\vec{r}) = N_2\varphi_s(r)Y_{20}(\theta, \varphi), \tag{6}$$

and the other components of the quadrupole field,

$$\bar{\varphi}_{2m}(\vec{r}) = N_2\varphi_s(r)Y_{2m}(\theta, \varphi), \tag{7}$$

with the normalization factor N_2:

$$\phi(\vec{r}) = \varphi_p(\vec{r}) + \hat{\phi}(\vec{r}), \tag{8}$$
$$\pi(\vec{r}) = 0, \tag{9}$$

where the quadrupole field is quantized as

$$\hat{\phi}(\vec{r}) = \sum_m \frac{1}{\sqrt{2\omega_2}}\{a_{2m}\bar{\varphi}_{2m}(\vec{r}) + a_{2m}^\dagger\bar{\varphi}_{2m}^*(\vec{r})\} \tag{10}$$

and

$$\omega_2^2 = \langle\bar{\varphi}_{2m}| - \nabla^2 + m^2|\bar{\varphi}_{2m}\rangle. \tag{11}$$

The spherical principal mean field $\varphi_p(\vec{r})$ is assumed to contribute only to the nuclear single-particle energies, but not to the residual interaction energies. The set of the quadrupole fields $\bar{\varphi}_{2m}$ make the meson field to be rotationally invariant. The components for $m \neq 0$ of the quadrupole field are necessary to take into account any orientation of the deformed Hartree states in the initial and final states of the tunneling process and to give rise to the collective tunneling transitions between the Hartree states.

The meson field ϕ in Eq. (8) is related to the nucleon field ψ by the field equation,

$$(-\nabla^2 + m^2)\phi(\vec{r}) = g\psi^\dagger(\vec{r})\psi(\vec{r}). \tag{12}$$

Projecting the above equation onto the quadrupole field, we express the quantum quadrupole meson field $\hat{\phi}$ in terms of the nucleon field:

$$\hat{\phi}(\vec{r}) = \frac{1}{\omega_2^2}\sum_m \bar{\varphi}_{2m}(\vec{r})\int \bar{\varphi}_{2m}^*(\vec{r'})(-\nabla'^2 + m^2)\hat{\phi}(\vec{r'})d^3r'$$
$$= \frac{g}{\omega_2^2}\sum_m \bar{\varphi}_{2m}(\vec{r})\int \bar{\varphi}_{2m}^*(\vec{r'})\hat{\psi}^\dagger(\vec{r'})\hat{\psi}(\vec{r'})d^3r', \tag{13}$$

where the nucleon field $\hat{\psi}(\vec{r})$ is quantized in terms of the creation and annihilation operators c_{si} for s nucleons and $c_{\mu i}$ for d_μ nucleons in the basis states determined in the single-particle potential for the spherical principal mean field $\varphi_p(\vec{r})$ in Eq. (4):

$$\hat{\psi}(\vec{r}) = \sum_i (c_{si}\psi_s(\vec{r}) + \sum_\mu c_{\mu i}\psi_\mu(\vec{r})). \tag{14}$$

The suffix μ stands for the z-component of the orbital angular momentum $l=2$ for d-state nucleons and the suffix i for spin and isospin of nucleons.

The expressions (8) and (12)-(13) for the meson field are substituted into the nuclear field Hamiltonian H_{nucl} in Eq. (1). This procedure for the nuclear field Hamiltonian determines the Hamiltonian effective for the Hartree states and also for the collective tunneling:

$$H_{\text{nucl}} = H_{\text{shell}} + H_{\text{int}}. \tag{15}$$

The quadrupole meson fields $\bar{\varphi}_{2m}(\vec{r})$ yield the nuclear quadrupole-quadrupole interaction, responsible for the two deformed Hartree states and also for the collective tunneling transitions between the two Hartree states,

$$H_{\text{int}} = -\frac{g^2}{2\omega_2^2} \int \int \sum_m \hat{\psi}^\dagger(\vec{r})\bar{\varphi}_{2m}(\vec{r})\hat{\psi}(\vec{r})\hat{\psi}^\dagger(\vec{r}')\bar{\varphi}_{2m}^*(\vec{r}')\hat{\psi}(\vec{r}')d^3r d^3r'. \tag{16}$$

The shell-model Hamiltonian H_{shell} for nucleons in the s and d shells is for the spherical shell model,

$$H_{\text{shell}} = \sum_i (\varepsilon_s c_{si}^\dagger c_{si} + \sum_\mu \varepsilon_d c_{\mu i}^\dagger c_{\mu i}). \tag{17}$$

When the valence nucleon configurations are truncated in the s and d major shell, the quadrupole-quadrupole interaction Hamiltonian H_{int} in Eq. (16), responsible for the deformed Hartree states and also for the collective tunneling, is divided into the two interactions, i.e., the interaction H between a pair of d-state nucleons and the interaction H_{sd} to excite d-state nucleons to s state and vice versa,

$$H_{\text{int}} = H + H_{sd}, \tag{18}$$

where

$$H = -\frac{g^2}{2\omega_2^2}\sum_{mm'ii'}u_{mm'}c_{m+m'i}^\dagger c_{-m-m'i'}^\dagger c_{-mi'}c_{mi} \tag{19}$$

$$H_{sd} = -\frac{g^2}{2\omega_2^2}\sum_{mii'}(v_m c_{mi}^\dagger c_{-mi'}^\dagger c_{si'}c_{si} + \text{H.c.}). \tag{20}$$

The quadrupole-quadrupole interaction H between a pair of d nucleons plus the shell-model Hamiltonian H_{shell} form the deformed Hartree potential for the prolate Hartree state and also that for the oblate Hartree state. On the other hand, the s-d interaction H_{sd}, which is spherically symmetric, yields the residual force for the prolate Hartree state to tunnel to the oblate Hartree state and for the return process in the present theory beyond the Hartree approximation. In the numerical calculations below, the single-particle energies ε_s and ε_d in the spherical shell-model Hamiltonian H_{shell} are assumed to be degenerate ($\varepsilon_s = \varepsilon_d$) and the spin-orbit splitting for the d states is ignored. In Section IV, we solve the secular equation for the nuclear

Hamiltonian H_{nucl} in Eq. (15) and express the collective tunneling transitions between the two Hartree states in the real-time description. Taking advantage of SU(5) symmetry in the quadrupole-quadrupole interaction Hamiltonian H, we solve the problems of the collective tunneling. In the present theory, the s-d interaction H_{sd} plays role of the pairing interaction [9, 10] in the collective tunneling between the deformed Hartree states.

III. SU(5) EXPRESSION FOR INTERACTION HAMILTONIAN H BETWEEN d-STATE NUCLEONS

A. SU(5) expression for Hamiltonian H

The nucleus of ^{28}Si, which has 12 nucleons in the s and d major shell, has a prolate and an oblate Hartree state. Suppose that the nucleus ^{28}Si is polarized along z-axis in either a prolate or an oblate Hartree state. When the spin-orbit interactions are neglected, the single-particle states $(d_1\ d_{-1}\ d_0)^4$ are occupied in the prolate state and $(d_2\ d_{-2}\ s)^4$ are occupied in the oblate state, where s and d_μ stand for the s and d single-particle states, respectively. The index 4 represents the degrees of freedom for spin and isospin for s and d nucleons.

The transformation generators in the space of the five single-particle states $d_{\pm 2}$, $d_{\pm 1}$ and d_0 constitute an SU(5) algebra. Actually the Hamiltonian H in Eq. (19) is expressed in terms of the quadrupole operators Q_m of the SU(5) generators, as is shown below. In this Subsection, we study the properties of the nuclear system governed by the Hamiltonian H in Eq. (19) in terms of SU(5). The 24 generators for the SU(5) algebra are expressed by the creation and annihilation operators $c_{\mu i}$ for the d-state

nucleons as the multi-pole operators:

$$L_m = \sqrt{10}\sum_{\mu i}(2\mu 2m-\mu|1m)(-1)^{\mu-m}c_{\mu i}^\dagger c_{\mu-mi} \quad (21)$$

$$Q_m = \sqrt{10}\sum_{\mu i}(2\mu 2m-\mu|2m)(-1)^{\mu-m}c_{\mu i}^\dagger c_{\mu-mi} \quad (22)$$

$$S_m = \sqrt{10}\sum_{\mu i}(2\mu 2m-\mu|3m)(-1)^{\mu-m}c_{\mu i}^\dagger c_{\mu-mi} \quad (23)$$

$$R_m = \sqrt{10}\sum_{\mu i}(2\mu 2m-\mu|4m)(-1)^{\mu-m}c_{\mu i}^\dagger c_{\mu-mi} \quad (24)$$

The z-component of the nuclear orbital angular momentum L,

$$L_0 = \sum_i(2c_{2i}^\dagger c_{2i}+c_{1i}^\dagger c_{1i}-c_{-1i}^\dagger c_{-1i}-2c_{-2i}^\dagger c_{-2i}), \quad (25)$$

and the nuclear quadrupole moment,

$$-Q_0 = -\sqrt{\frac{5}{7}}\sum_i \left(\; 2c_{2i}^\dagger c_{2i}-c_{1i}^\dagger c_{1i}-2c_{0i}^\dagger c_{0i} \right.$$
$$\left. -\; c_{-1i}^\dagger c_{-1i}+2c_{-2i}^\dagger c_{-2i}\right), \quad (26)$$

are additive collective quantities. The operators Q_m and $L_{m'}$ satisfy the commutation relation,

$$[Q_m,L_{m'}] = -\sqrt{6}(2m1m'|2m+m')Q_{m+m'}, \quad (27)$$

which is identical to that for SU(3) by Elliott [11]. The commutation relation of Q_m with $Q_{m'}$, however, involves not only $L_{m+m'}$ so as the case of SU(3) but also octupole operators $S_{m+m'}$. Thus, the algebra of the 24 transformation generators $c_{\mu i}^\dagger c_{\mu' i}$ that appear in the Hamiltonian H in Eq. (19) is extended to SU(5) [12], which employs, in addition to L_m and Q_m, the octupole and hexadecapole operators, S_m and R_m. The commutation relations of these operators for SU(5) algebra are expressed in terms of $6j$ symbols: If we use the unified expression for these multi-pole operators,

$$O_{lm} = \begin{cases} L_m, & \text{for} \quad l=1, \\ Q_m, & \text{for} \quad l=2, \\ S_m, & \text{for} \quad l=3, \\ R_m, & \text{for} \quad l=4, \end{cases} \quad (28)$$

the commutation relations are

$$[O_{l_1m_1},O_{l_2m_2}] = \sum_{l(=l_1+l_2+\text{odd})}(-1)^l 2\sqrt{10(2l_1+1)(2l_2+1)}\begin{Bmatrix} 2 & 2 & l \\ l_1 & l_2 & 2 \end{Bmatrix}$$
$$\times(l_1m_1l_2m_2|lm_1+m_2)O_{lm_1+m_2}. \quad (29)$$

The Casimir operator for SU(5) is defined as

$$C = L^\dagger\cdot L+Q^\dagger\cdot Q+S^\dagger\cdot S+R^\dagger\cdot R. \quad (30)$$

The quadrupole-quadrupole interaction Hamiltonian H between a pair of d nucleons in Eq. (19) is expressed in terms of the quadrupole operators Q_m as

$$H = -\kappa Q^\dagger\cdot Q = -\kappa\sum_{m=-2}^2 Q_m^\dagger Q_m \quad (31)$$

and the interaction Hamiltonian H_{sd} to excite d nucleons to s state and vice versa,

$$H_{sd} = -\frac{\kappa'}{2\sqrt{6}}\sum_{mii'}(-1)^m\{c_{mi}^\dagger c_{-mi'}^\dagger c_{si'}c_{si}+\text{H.c.}\}. \quad (32)$$

The quadrupole operators conjugate to Q_m are defined to be

$$Q_m^\dagger = (-1)^m Q_{-m}. \quad (33)$$

The coupling constants κ and κ' of the quadrupole-quadrupole interactions H and H_{sd} in Eq's. (31) and (32) are determined in terms of the meson quadrupole mean field $\bar\varphi_{2m}$ overlapped with the nucleon wave functions:

$$\kappa = \frac{g^2}{20\omega_2^2}\sum_\mu\int\int\psi_{2\mu-m}^*(\vec{r})\bar\varphi_{2m}^*(\vec{r})\psi_{2\mu}(\vec{r})\psi_{2-\mu+m}^*(\vec{r'})\bar\varphi_{2m}(\vec{r'})\psi_{2-\mu}(\vec{r'})d^3rd^3r',$$
$$\kappa' = \frac{\sqrt{6}g^2}{\omega_2^2}\int\int\psi_{2m}^*(\vec{r})\bar\varphi_{2m}(\vec{r})\psi_s(\vec{r})\psi_{2-m}^*(\vec{r'})\bar\varphi_{2-m}(\vec{r'})\psi_s(\vec{r'})d^3rd^3r'. \quad (34)$$

The values of κ and κ' that are determined in the Hartree calculations are of the order of 0.1 MeV for the s-d shell nuclei [6]. In the numerical calculation in Section IV, we assume that $\kappa = \kappa'$.

The present interaction Hamiltonian $H = -\kappa Q^\dagger \cdot Q$ in Eq. (31) commutes with the Casimir operator C,

$$[H, C] = 0. \tag{35}$$

Therefore, eigenstates of the Hamiltonian H are concurrently eigenstates of the Casimir operator C. Eigenstates of the Casimir operator C with a same eigenvalue constitute a representation of SU(5). Any eigenstates of the interaction Hamiltonian H can be classified by representations of SU(5) with a given eigenvalue of the Casimir operator C. Since the Hamiltonian H is rotational symmetric, we obtain

$$[H, L_0] = 0, \quad [H, L_{\pm 1}] = 0, \quad [H, \vec{L}^2] = 0 : \tag{36}$$

The eigenstates of the Hamiltonian H are simultaneously eigenstates $|L, M\rangle$ of the orbital angular momentum operators $\vec{L}^2$ and L_0. The eigenvalues of the Hamiltonian H depend on the representation of SU(5) and on the angular momentum L but not on M.

In the previous paper [2], applying the SU(5) Hamiltonian H in Eq. (31) to some typical systems of two and four d-state nucleons, we demonstrated that the Hamiltonian H has many appropriate aspects for realistic systems, where we solved the eigenequations for H and C,

$$H\Psi_{(k)} = \varepsilon_k \Psi_{(k)}, \tag{37}$$
$$C\Psi_{(k)} = \lambda_k^2 \Psi_{(k)}. \tag{38}$$

Antisymmetrized states of two d nucleons with a unique combination of spin and isospin such as two protons with spin up, which have nuclear orbital angular momentum $L = 1$ or 3, are classified into the 10-dimensional representation of SU(5).

B. Representations of SU(5) for nuclear states of ^{28}Si

The SU(5) Hamiltonian $H = -\kappa Q^\dagger \cdot Q$ in Eq. (31) yields the deformed Hartree states of an s-d major-shell nucleus: The Hartree states of the nucleus ^{28}Si, which has 12 valence nucleons in the s-d major shell, are determined by the Hartree Hamiltonian for H,

$$H_{\text{Hart}} = -2\kappa\langle Q_0\rangle Q_0 + \kappa\langle Q_0\rangle^2. \tag{39}$$

Since the Hartree Hamiltonian H_{Hart} is independent of the spin and isospin of nucleons, the nuclear wave functions for the nucleons in the s-d major shell are expressed by a product of the four antisymmetrized wave functions, i.e., that for three protons with spin up, that for three protons with spin down, that for three neutrons with spin up and that for three neutrons with spin down.

In the oblate Hartree state of the nucleus ^{28}Si polarized along z-axis, the single-particle states $(d_2 \, d_{-2} \, s)^4$ are occupied. On the whole basis set of nuclear oblate states in the nucleon configurations $d^8 s^4$, which have 8 nucleons of the 12 valence nucleons in the d states and 4 nucleons in the s state, we diagonalize the Hamiltonian H in Eq. (31) to determine the nuclear oblate states with a given angular momentum L. The 12-nucleon system has 3 nucleons, i.e., two d nucleons and one s nucleon, for each a unique combination of spin and isospin of nucleons, such as protons with spin up, for example. The orbital wave functions of the three nucleons for a unique combination of spin and isospin are antisymmetrized with the compound orbital angular momentum $L = 1$ or 3. The oblate basis states for the 12-nucleon system are formulated by a product of the four antisymmetrized wave functions for each a set of three nucleons with a unique combination of spin and isospin. Since the antisymmetrized wave functions of two d nucleons and one s nucleon with a unique combination of spin and isospin are expressed by the 10-dimensional representation of SU(5) as mentioned at the end of Subsection A, the nuclear states of 12 nucleons in the nucleon configurations $d^8 s^4$ are classified by the representations of SU(5) that are formed by a product $10 \times 10 \times 10 \times 10$ of SU(5) (see Table I(a)). The Hamiltonian $H = -\kappa Q^\dagger \cdot Q$ in Eq. (31) is diagonalized in the space of the nuclear basis states in the 490-dimensional representation formed by the product $10 \times 10 \times 10 \times 10$ for the nucleon configurations $d^8 s^4$. The ground state of the Hamiltonian H in the nucleon configurations $d^8 s^4$ is the lowest-lying state in the 490-dimensional representation. This state is identified as the ground state of oblate states with the nuclear angular momentum $L = 0$. The 490-dimensional representation exhibits the rotational ground band in the space of the nucleon configurations $d^8 s^4$ for the Hamiltonian H. The structure of the 490-dimensional representation is shown by the Young tableau in Fig. 1(a).

On the other hand, in the prolate Hartree state polarized along z-axis, the single-particle states $(d_1 \, d_{-1} \, d_0)^4$ are occupied: The nucleus has the 12 valence nucleons in the d states. On the whole basis set of nuclear prolate states in the nucleon configurations d^{12}, which have all the 12 valence nucleons in the d states, we diagonalize the Hamiltonian H in Eq. (31) to determine the nuclear prolate states with a given angular momentum L. The prolate basis states for the 12 nucleons in the d states are expressed by a product of the four antisymmetrized orbital wave functions for three d nucleons with a unique combination of spin and isospin: Nuclear prolate states of 12 nucleons in the nucleon configurations d^{12} are classified by the representations of a product $10^* \times 10^* \times 10^* \times 10^*$ of the 10^*-dimensional representations for the antisymmetrized wave functions of three d nucleons with a unique combination of spin and isospin. These SU(5) representations for the nucleon configurations d^{12} are conjugate to the representations for the nucleon configurations $d^8 s^4$. The ground state of the Hamil-

tonian H in the configurations d^{12} is the lowest-lying state in the 490*-dimensional representation formed by the product $10^* \times 10^* \times 10^* \times 10^*$ of the 10*-dimensional representations. This state is identified as the ground state of prolate states with the nuclear angular momentum $L = 0$. The energy spectrum for the Hamiltonian H for the nucleon configurations d^{12} in the space of the 490*-dimensional representation is of a same structure as the energy spectrum for the nucleon configurations $d^8 s^4$ in the space of the 490-dimensional representation: The 490*-dimensional representation exhibits the rotational ground band in the space of the nucleon configurations d^{12} for the Hamiltonian H in Eq. (31). The structure of the 490*-dimensional representation is shown by the Young tableau in Fig. 1(b).

Although the Hamiltonian H in Eq. (31) does not couple the prolate states with the oblate states, the s-d interaction H_{sd} does in second-order processes. Therefore, we introduce the intermediate nuclear states in the nucleon configurations $d^{10} s^2$ which couple the 490-dimensional oblate states in the configurations $d^8 s^4$ with the 490*-dimensional prolate states in the configurations d^{12}. The significant nuclear states in the configurations $d^{10} s^2$ intermediate between the prolate and oblate states are in the representations of SU(5) that are formed by a product $10^* \times 10^* \times 10 \times 10$ of the 10- and 10*-dimensional representations for three nucleons in the configurations $d^2 s$ and d^3, respectively, for a unique combination of spin and isospin (see Table I(b)). We diagonalize the Hamiltonian H on the basis states in the nucleon configurations $d^{10} s^2$. The lowest-lying eigenenergy state of the Hamiltonian H in the nucleon configurations $d^{10} s^2$ is classified into the 1176-dimensional representation. The 1176-dimensional representation exhibits the rotational ground band in the space of the nucleon configurations $d^{10} s^2$ for the Hamiltonian H. The structure of the 1176-dimensional representation is shown by the Young tableau in Fig. 1(c).
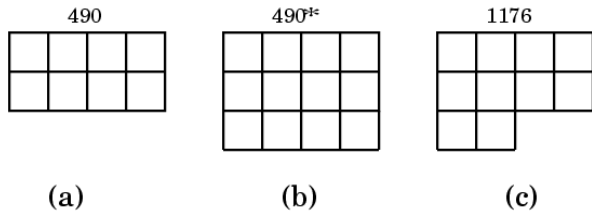

FIG. 1: The Young tableaus for the (a) 490-, (b) 490*- and (c) 1176-dimensional representations of SU(5). Each one box represents a d-state nucleon.

The eigenstates of H are classified by SU(5) representations. In Fig.2, we show the eigenenergies of H in Eq. (31) in the three representations, i.e., the (a) 490- and 490*-, and (b) 1176-dimensional representations. We see that the ground state band features a rotational structure of the nuclear energy levels in each a representation.
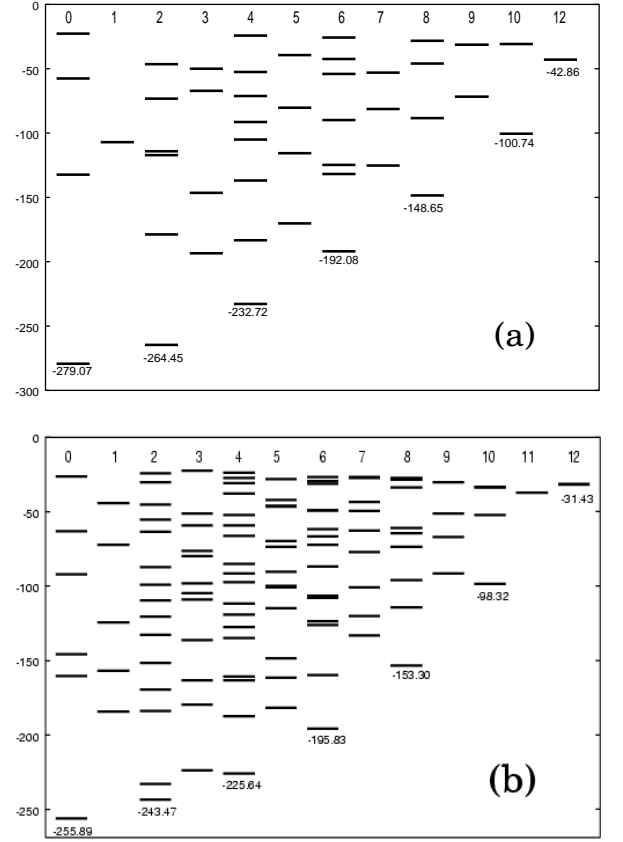

FIG. 2: The energy spectra for (a) the 490-dimensional representation and for (b) the 1176-dimensional representation. The energy spectrum for the 490*-dimensional representation is of a same structure as that for the 490-dimensional representation. The energies of the nuclear levels are shown in unit of κ. The integral numbers shown at the top of the figures stand for the angular momenta L of the nuclear states.

IV. COLLECTIVE TUNNELING TRANSITIONS BETWEEN DEFORMED HARTREE STATES

The s-d interaction Hamiltonian H_{sd} couples a nuclear oblate state in the 490-dimensional representation with a prolate state in the 490*-dimensional representation through an intermediate state in the 1176-dimensional representation as is shown in Fig. 3: Nuclear tunneling transitions from an oblate state to a prolate state proceed through an intermediate state, hopping in a two-step process [13]. Taking into account the Hamiltonian H_{sd}, we solve the secular equation for the nuclear Hamiltonian H_{nucl} in Eq. (15),

$$H_{\text{nucl}} \Psi_k = E_k \Psi_k, \tag{40}$$

in the combined space of the nuclear states in the 490-, 1176- and 490*-dimensional representations of SU(5). The eigenstates of the Hamiltonian H_{nucl}, which commutes with the angular momentum operators L_m, are

TABLE I: The nuclear states of the system of 12 nucleons in the configurations (a) d^8s^4 and (b) $d^{10}s^2$ are classified by the representations of SU(5). The nuclear states of 12 nucleons in the configurations d^{12} are classified into the representations conjugate to those for the configurations d^8s^4. The dimension and eigenvalue λ^2 of the Casimir operator C for each a representation are shown. The column headed as Multiplicity shows the multiplicity of the representations with a same dimension and eigenvalue λ^2. Each a representation contains the nuclear states specified by their angular momenta L with the multiplicities M, which are shown as L^M.

(a) The nuclear states in the nucleon configurations d^8s^4.

Dimension	λ^2	Multiplicity	L^M
1050	352	3	11 10^2 9^4 8^6 7^{10} 6^{12} 5^{15} 4^{15} 3^{16} 2^{12} 1^9 0^2
560	312	2	10 9 8^4 7^4 6^8 5^7 4^{11} 3^7 2^{10} 1^3 0^4
490	432	1	12 10^2 9^2 8^4 7^3 6^7 5^4 4^8 3^4 2^6 1 0^4
450	272	3	9 8^2 7^4 6^5 5^8 4^8 3^9 2^6 1^6 0
315	272	3	9 8 7^3 6^3 5^6 4^5 3^7 2^4 1^5
210	232	6	8 7 6^3 5^3 4^5 3^4 2^5 1^2 0^2
175	192	7	7 6^2 5^3 4^4 3^5 2^4 1^3 0
40	132	8	5 4 3 2^2 1
35	192	1	6 4 3 2 0
15	112	3	4 2 0
10	72	6	3 1

(b) The nuclear states in the nucleon configurations $d^{10}s^2$.

Dimension	λ^2	Multiplicity	L^M
176	400	1	12 11 10^4 9^4 8^9 7^9 6^{15} 5^{13} 4^{18} 3^{12} 2^{15} 1^5 0^6
1024	300	4	10 9^3 8^5 7^9 6^{13} 5^{15} 4^{18} 3^{18} 2^{14} 1^{10} 0^4
700	360	1	11 10 9^3 8^4 7^7 6^7 5^{11} 4^9 3^{11} 2^7 1^7
700*	360	1	11 10 9^3 8^4 7^7 6^7 5^{11} 4^9 3^{11} 2^7 1^7
200	240	2	8 7 6^3 5^3 4^5 3^3 2^5 1 0^2
175	240	3	8 7 6^2 5^3 4^4 3^3 2^4 1^2 0
175*	240	3	8 7 6^2 5^3 4^4 3^3 2^4 1^2 0
126	200	4	7 6 5^3 4^2 3^4 2^2 1^3
126*	200	4	7 6 5^3 4^2 3^4 2^2 1^3
75	160	9	6 5 4^2 3^2 2^3 1 0
24	100	8	4 3 2 1
1	0	3	0

simultaneously eigenstates of the angular momentum. Since the oblate Hartree states in the configurations d^8s^4 are degenerate one to one correspondingly to the prolate Hartree states in the configurations d^{12}, there appear two almost degenerate lowest-lying nuclear states for each of angular momenta $L = 0$, 2 and 4. The calculated energy splitting ΔE between the almost degenerate pair of the ground state and the first excited state for $L=0$, 2 and 4 are shown in Table II. We obtain approximate values ΔE=1.37, 0.94 and 1.00 of the energy splitting in unit of κ for $L = 0$, 2 and 4 in the simplified calculations which use only one lowest-lying state for each of the three sets of configurations d^8s^4, $d^{10}s^2$ and d^{12} [14]. The perturbational expression for the energy splitting for the three-state system,

$$\Delta E = \frac{2h^2}{\varepsilon_0^{1176} - \varepsilon_0^{490}}, \qquad (41)$$

yields a reasonablly approximate value. In the above expression, ε_0^{490} and ε_0^{1176} stand for the energies of the ground states $\Psi_{(0)}^{490}$ and $\Psi_{(0)}^{1176}$ for given L in the 490- (490*-) and 1176-dimensional representation spaces, respectively, and h is the matrix element of the interac-

TABLE II: The energies E_k of the nuclear ground state and first excited state for angular momentum $L = 0$, 2 and 4 and their splitting ΔE calculated in Eq. (40) are shown in unit of κ.

L	Ground State	First Excited State	ΔE
0	-280.47	-279.14	1.33
2	-265.46	-264.71	0.75
4	-233.87	-232.94	0.93

tion Hamiltonian H_{sd} between the ground states of the Hamiltonian H:

$$h = \langle \Psi_{(0)}^{1176} | H_{sd} | \Psi_{(0)}^{490} \rangle.$$

Now, we formulate the collective tunneling transitions between prolate and oblate Hartree states. In a real-time description, the probability of the collective tunneling transitions from an initial state Ψ_i to a final state Ψ_f is expressed in terms of the eigenstates Ψ_k and eigenenergies E_k of the nuclear Hamiltonian H_{nucl} determined
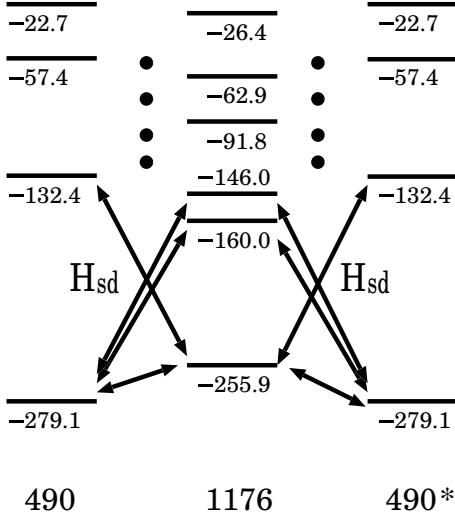
FIG. 3: The processes of the tunneling transitions between the nuclear oblate states in the 490-dimensional representation and the nuclear prolate states in the 490*-dimensional representation with angular momentum $L = 0$ are illustrated. The tunneling transitions proceed through nuclear intermediate states in the 1176-dimensional representation in terms of the Hamiltonian H_{sd}.

in Eq. (40) as

$$
\begin{aligned}
P_{fi}(t) &= |\langle \Psi_f | e^{-i H_{\mathrm{nucl}} t} | \Psi_i \rangle|^2 \\
&= |\sum_k \langle \Psi_f | \Psi_k \rangle e^{-i E_k t} \langle \Psi_k | \Psi_i \rangle|^2. \quad (42)
\end{aligned}
$$

We have performed two kinds of the calculations. First, we calculated the probabilities of the nuclear collective tunneling transitions for given orbital angular momentum L from the prolate ground state in the 490*-dimensional representation space for the configurations d^{12} to the oblate ground state in the 490-dimensional representation space for the configurations $d^8 s^4$. Since the Hamiltonian H_{nucl} is rotationally symmetric, the transitions between the two ground states conserve the orbital angular momentum L. The calculated transition probabilities between the nuclear ground states for the orbital angular momentum $L = 0$, 2 and 4 are shown in Fig. 4. We see that each of the transition probabilities features a gross structure of the harmonic collective tunneling oscillations perturbed by small quantum mechanical fluctuations.

Secondly, we calculated the probabilities of the tunneling transitions from a prolate Hartree state to an oblate Hartree state. Hereafter, the prolate Hartree state polarized along z-axis is called z-prolate state. We calculate the probability in the two cases of the tunneling transitions starting with the z-prolate state to a final oblate Hartree state. One is the case of the tunneling to the oblate Hartree state polarized along z-axis (z-oblate state) and the other is to a linear combination of the two oblate Hartree states, i.e., the oblate state polarized
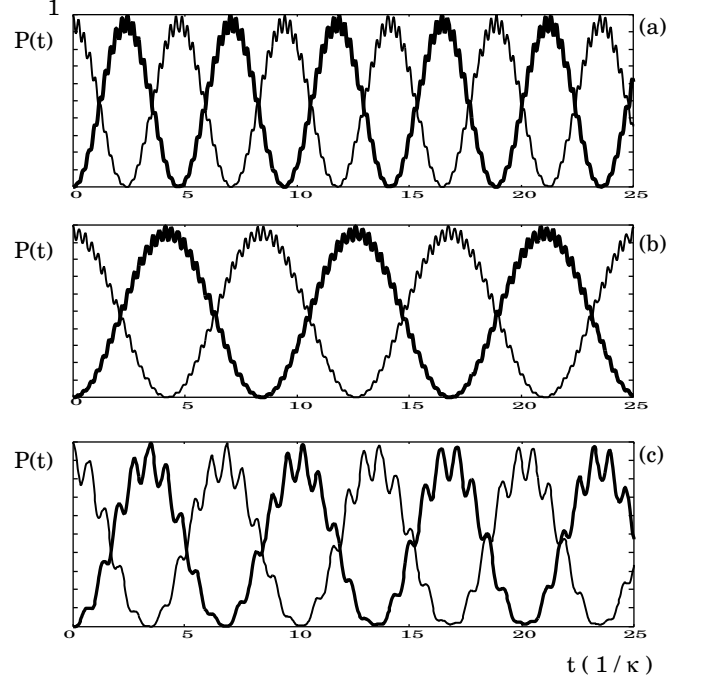

FIG. 4: The calculated probability of the tunneling transitions from the prolate ground state to the oblate ground state of the nucleus with angular momentum (a) $L = 0$, (b) 2 and (c) 4. The probability of the return transitions to the initial prolate ground state is also shown in Figures (a) to (c). The transition probabilities feature grossly the harmonic oscillation of the nucleus between the prolate and oblate states. The frequency of the harmonic oscillation is determined by the energy splitting ΔE between the ground state and the first excited state of the nuclear Hamiltonian H_{nucl} for given angular momentum L.

along x-axis (x-oblate state) and the oblate state polarized along y-axis (y-oblate state),

$$
|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|x-\text{oblate}\rangle + |y-\text{oblate}\rangle). \quad (43)
$$

We show the calculated transition probabilities in Fig. 5. We see that the transition probability from the z-prolate state to the x- and y-oblate states is suppressed by about 40 percent, compared with the transition probability from the z-prolate state to the z-oblate state.

V. NUCLEAR TUNNELING TRANSITIONS THROUGH TRI-AXIALLY ASYMMETRIC DEFORMED STATES

We have formulated the nuclear collective tunneling transitions between a prolate Hartree state and an oblate Hartree state, taking into account the angular momentum conservation. The deformed Hartree states may be polarized along any axis. We may consider any "twisted"
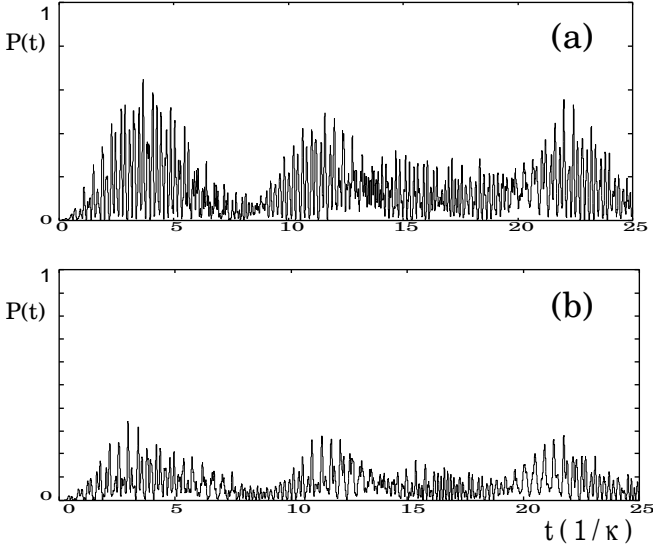
FIG. 5: The calculated probabilities of the tunneling transitions from a prolate Hartree state to an oblate Hartree state. We show the probabilities of the transitions from z-prolate Hartree state to (a) z-oblate Hartree state and to (b) the linear combination of x- and y-oblate states shown in Eq. (43).

tunneling transitions, such as that from the z-prolate state to the x-oblate state.

In the course of a tunneling transition from z-prolate state to z-oblate state, if the nucleus keeps rotational symmetry around z-axis in the shape, the nucleus proceeds through a spherical intermediate state with quadrupole moment $\langle -Q_0 \rangle = 0$ lying at the top of a high potential energy barrier. The high potential energy barrier on the way suppresses the transition probability. We will discuss in this Section that in the twisted collective tunneling transitions, such as the transitions from z-prolate state either to x-oblate state or to y-oblate state, the nucleus proceeds through tri-axially asymmetric deformed intermediate states [3][4], i.e., through a lower potential energy barrier than the spherical potential energy barrier in the course of the direct tunneling transition from z-prolate state to z-oblate state.

Hereafter we express the z-prolate and z-oblate Hartree states with spin and isospin $S = T = 0$ explicitly in terms of occupied single-particle states as

$$|z-\text{prolate}\rangle = |(d_1 \ d_{-1} \ d_0)^4\rangle, \qquad (44)$$
$$|z-\text{oblate}\rangle = |(d_2 \ d_{-2} \ s)^4\rangle. \qquad (45)$$

Rotating the above z-prolate and z-oblate states by 90 degrees around y-axis, we obtain the x-prolate and x-oblate states, respectively, which are expressed in terms of occupied single-particle states as

$$|x-\text{prolate}\rangle = |\{\frac{1}{\sqrt{2}}(d_2 - d_{-2}) \ \frac{1}{\sqrt{2}}(d_1 - d_{-1}) \ (-\frac{1}{2}d_0 + \frac{\sqrt{3}}{2\sqrt{2}}(d_2 + d_{-2}))\}^4\rangle,$$

$$(46)$$

$$|x-\text{oblate}\rangle = |\{\frac{1}{\sqrt{2}}(d_1 + d_{-1}) \ (\frac{\sqrt{3}}{2}d_0 + \frac{1}{2\sqrt{2}}(d_2 + d_{-2})) \ s\}^4\rangle. \qquad (47)$$

We also obtain the y-prolate and y-oblate states, rotating the z-prolate and z-oblate states, respectively, by -90 degrees around x-axis,

$$|y-\text{prolate}\rangle = |\{\frac{1}{\sqrt{2}}(d_2 - d_{-2}) \ \frac{1}{\sqrt{2}}(d_1 + d_{-1}) \ (\frac{1}{2}d_0 + \frac{\sqrt{3}}{2\sqrt{2}}(d_2 + d_{-2}))\}^4\rangle,$$

$$(48)$$

$$|y-\text{oblate}\rangle = |\{\frac{1}{\sqrt{2}}(d_1 - d_{-1}) \ (-\frac{\sqrt{3}}{2}d_0 + \frac{1}{2\sqrt{2}}(d_2 + d_{-2})) \ s\}^4\rangle. \qquad (49)$$

Note that the Hartree energies of these six deformed states are degenerate.

From the above nucleon configurations (44)-(49) of Hartree states of the s and d major-shell nucleus, we see that the direct tunneling transitions from z-prolate state to z-oblate state take place, only in the case that all of

the 12 nucleons in d_1, d_{-1} and d_0 states are excited to d_2, d_{-2} and s states. The twisted tunneling transitions can, however, take place by exciting only four nucleons in a second order process of the s-d interaction H_{sd}. The twisted tunneling transitions from z-prolate state to x-oblate state, for example, can take place by exciting four nucleons in $\frac{1}{\sqrt{2}}(d_1 - d_{-1})$ state to s state in a second order process of the s-d interaction H_{sd}, where the nucleons in the states $\frac{1}{\sqrt{2}}(d_1 + d_{-1})$ and d_0 are spectators, which are not affected by the interaction Hamiltonian H_{sd}.

The second-order processes in H_{sd} in the nuclear collective tunneling transitions from z-prolate state to x-oblate state proceed through intermediate states. One of the typical intermediate states for the tunneling transitions between the two deformed Hartree states is obtained by a square root of a product of the two Hartree states,

$$|\{(z-\text{prol})(x-\text{obl})\}^{\frac{1}{2}}\rangle = |\{\frac{1}{\sqrt{2}}(d_1 + d_{-1})\}^4$$
$$\{d_0\,(\frac{\sqrt{3}}{2}d_0 + \frac{1}{2\sqrt{2}}(d_2 + d_{-2}))\}^2\,\{\frac{1}{\sqrt{2}}(d_1 - d_{-1})\,s\}^2\rangle, \tag{50}$$

where the 4 nucleons expressed by each a pair of the parentheses { and } constitute a spatial symmetric state with spin and isospin $S = T = 0$. This state is a tri-axially asymmetric deformed state with $\gamma=30$ where γ is defined [3] as

$$\tan\gamma = -\frac{\langle Q_2 \rangle + \langle Q_{-2} \rangle}{\sqrt{2}\langle Q_0 \rangle}. \tag{51}$$

Note that we define the angle γ along the direction opposite to that in Ref. [3]. The transition matrix elements for the second-order tunneling process from z-prolate state to x-oblate state through this intermediate state are

$$\langle \{(z-\text{prol})(x-\text{obl})\}^{\frac{1}{2}}|H_{sd}|z-\text{prolate}\rangle = -\frac{9}{2}\sqrt{\frac{8}{219}}\kappa', \tag{52}$$

$$\langle x-\text{oblate}|H_{sd}|\{(z-\text{prol})(x-\text{obl})\}^{\frac{1}{2}}\rangle = -\frac{9}{2}\sqrt{\frac{8}{219}}\kappa'. \tag{53}$$

Similarly the second-order processes in H_{sd} in the nuclear collective tunneling transitions from z-prolate state to y-oblate state proceed through intermediate states. One of the typical intermediate states for the tunneling transitions between the two deformed Hartree states is obtained by a square root of a product of the two Hartree states,

$$|\{(z-\text{prol})(y-\text{obl})\}^{\frac{1}{2}}\rangle = |\{\frac{1}{\sqrt{2}}(d_1 - d_{-1})\}^4$$
$$\{d_0\,(-\frac{\sqrt{3}}{2}d_0 + \frac{1}{2\sqrt{2}}(d_2 + d_{-2}))\}^2\,\{\frac{1}{\sqrt{2}}(d_1 + d_{-1})\,s\}^2\rangle, \tag{54}$$

where the 4 nucleons expressed by each a pair of the parentheses { and } constitute a spatial symmetric state with spin and isospin $S = T = 0$. This state is a tri-axially asymmetric deformed state with $\gamma = -30$. The transition matrix elements for the second-order tunneling process from z-prolate state to y-oblate state through this intermediate state are

$$\langle \{(z-\text{prol})(y-\text{obl})\}^{\frac{1}{2}}|H_{sd}|z-\text{prolate}\rangle = \frac{9}{2}\sqrt{\frac{8}{219}}\kappa', \tag{55}$$

$$\langle y-\text{oblate}|H_{sd}|\{(z-\text{prol})(y-\text{obl})\}^{\frac{1}{2}}\rangle = \frac{9}{2}\sqrt{\frac{8}{219}}\kappa'. \tag{56}$$

The six Hartree states and six tri-axially asymmetric deformed intermediate states are schematically plotted in the β-γ plane in Fig. 6.
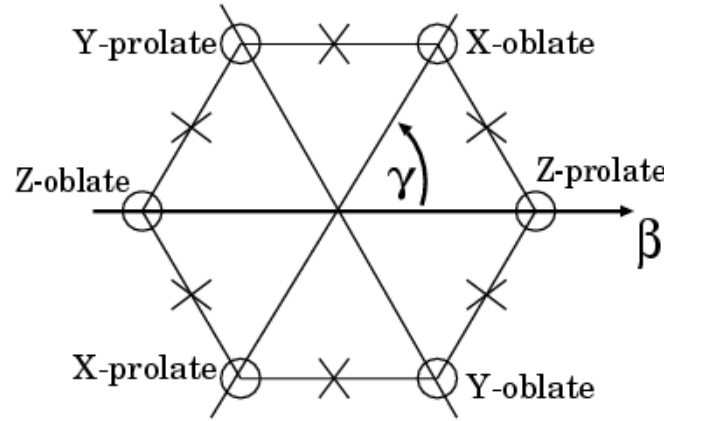

FIG. 6: The prolate Hartree states and oblate Hartree states polarized along z-, x- and y-axis are plotted with a circle and the intermediate states such as $|\{(x-\text{obl})(y-\text{prol})\}^{\frac{1}{2}}\rangle$ in the twisted tunneling transitions are also plotted with a cross in the β-γ plane.

While the spherical intermediate states in the tunneling transitions of the nucleus ^{28}Si are highly excited, the above intermediate states of the twisted collective tunneling transitions, which are tri-axially asymmetric deformed states, are not so highly excited. The twisted tunneling transitions through a lower potential energy barrier are not so much hindered as the direct tunneling transitions from z-prolate state to z-oblate state through a spherical intermediate state. Therefore we see that the tunneling transitions from z-prolate state to z-oblate state may take place mainly proceeding through some tri-axially asymmetric deformed intermediate states by several steps avoiding the direct tunneling through a spherical intermediate state:

$$|z-\text{prolate}\rangle \rightarrow$$

$$\left\{ \begin{array}{lll} |\{(z-\text{prol})(y-\text{obl})\}^{\frac{1}{2}}\rangle & |\{(y-\text{obl})(x-\text{prol})\}^{\frac{1}{2}}\rangle & |\{(x-\text{prol})(z-\text{obl})\}^{\frac{1}{2}}\rangle \\ |\{(z-\text{prol})(x-\text{obl})\}^{\frac{1}{2}}\rangle & |\{(x-\text{obl})(y-\text{prol})\}^{\frac{1}{2}}\rangle & |\{(y-\text{prol})(z-\text{obl})\}^{\frac{1}{2}}\rangle \end{array} \right\}$$

$$\rightarrow |z-\text{oblate}\rangle. \tag{57}$$

TABLE III: The coefficients c_{LM}^n of the normalized states of $H_{sd}|z-\text{oblate}\rangle$ and of $H_{sd}|z-\text{prolate}\rangle$ expanded in terms of the eigenstates of the Hamiltonian H with angular momentum L and M, and principal quantum number n in the 1176-dimensional representation. The presently concerned values of the coefficients c_{LM}^n only for $n = 1$ and $M = 0$ are shown. The double signs $\pm$ of the coefficient values correspond to the $|z-\text{prolate}\rangle$ and $|z-\text{oblate}\rangle$ states to be operated by H_{sd}, respectively.

L M	c_{L0}^1
0 0	0.32
2 0	±0.52
4 0	-0.41
6 0	±0.29
8 0	0.17
10 0	±0.07
12 0	-0.02

The tunneling transitions through tri-axially asymmetric deformed intermediate states lying on a lower potential energy barrier contribute to enhance the probability of the tunneling transitions from z-prolate state to z-oblate state. The tri-axially asymmetric deformed states such as $|\{(z-\text{prol})(y-\text{obl})\}^{\frac{1}{2}}\rangle$ belong to the 1176-dimensional representation of SU(5). They are main components of the rotational ground band states in the representation and are also main components of $H_{sd}|z-\text{prolate}\rangle$, as is shown in Table III. The numerical result reported in Section IV for the enhanced probability of the tunneling transitions from z-prolate state to z-oblate state reflects the significant contributions of various twisted tunneling processes through tri-axially asymmetric deformed intermediate states shown in Eq. (57).

The Hartree states defined in Eq's. (44)-(49) are expanded in terms of the eigenstates of the Hamiltonian H with angular momentum L and M, and principal quantum number n in the 490- (490*-) dimensional representation so as

$$|z-\text{prolate}\rangle = \sum_{nL} c_{L0}^n |L, M = 0, n\rangle, \tag{58}$$

$$|z-\text{oblate}\rangle = \sum_{nL} c_{L0}^n |L, M = 0, n\rangle. \tag{59}$$

The calculated coefficients c_{L0}^1 are shown in Table IV. While the Hartree states polarized along z-axis in Eq's. (44) and (45) are composed of only $M = 0$ components

TABLE IV: The coefficients c_{LM}^n of the prolate and oblate Hartree states in Eq's. (44)-(49) expanded in terms of the eigenstates of the Hamiltonian H with angular momentum L and M, and principal quantum number n in the 490- (490*-) dimensional representation. The presently concerned values of the coefficients c_{LM}^n only for $n = 1$ and $M = 0$ are shown. The summed probabilities $\sum_L (c_{L0}^1)^2$ for the Hartree states are also shown.

L M	$\begin{array}{c}\|z-\text{oblate}\rangle \\ \|z-\text{prolate}\rangle\end{array}$	$\begin{array}{c}\|x-\text{oblate}\rangle \\ \|y-\text{oblate}\rangle \\ \|x-\text{prolate}\rangle \\ \|y-\text{prolate}\rangle\end{array}$
0 0	0.34	0.34
2 0	-0.64	0.32
4 0	0.56	0.21
6 0	0.32	-0.10
8 0	0.13	0.04
10 0	0.04	-0.01
12 0	0.01	0.002
$\sum_L (c_{L0}^1)^2$	0.96	0.27

of the angular momentum L, the Hartree states polarized either along x-axis or along y-axis in Eq's. (46)-(49) have $M \neq 0$ components, with comparatively small $M = 0$ components as is shown in Table IV. Since the angular momentum of the nucleus is conserved in the course of the tunneling transitions, the probability of the twisted tunneling transitions from z-prolate state to the combination of x- and y-oblate states in Eq. (43) as the final state is suppressed by about 40 percent compared with that of the tunneling transitions from z prolate state to z-oblate state, as is seen in the numerical result in Section IV.

VI. DISCUSSIONS AND CONCLUSION

Using the physical quantities obtained in the Hartree calculations for the deformed Hartree states, we determine the Hamiltonian effective not only for the prolate and oblate Hartree states but also for the collective tunneling transitions between the two Hartree states. In the case of the nucleus of ^{28}Si, which has a prolate Hartree state and an oblate Hartree state symmetrically with respect to the deformation parameter, the nuclear field Hamiltonian H_{nucl} in Eq. (1) yields

the Hamiltonian H for the interaction between d nucleons and the Hamiltonian H_{sd} to excite d nucleons to s state and vice versa. The interaction Hamiltonian H between d nucleons, which we have expressed in terms of the SU(5) algebra, yields the nuclear deformed Hartree states. The Hamiltonian H_{sd} instead gives rise to the collective tunneling transitions between prolate states and oblate states. We have calculated the probabilities of the tunneling transitions between a prolate state and an oblate state in the real-time description.

The spherically symmetric nuclear field Hamiltonian H_{nucl} in Eq. (1) yields not only the interaction Hamiltonian H between d nucleons in Eq. (19) but also the residual s-d interaction H_{sd} in Eq. (20) for a deformed s and d shell nucleus in the present theory beyond the Hartree approximation. The rotationally symmetric Hamiltonian H in Eq. (19) recovers the rotational symmetry of the nuclear system by giving rise to the rotation of the deformed nucleus. Furthermore, the spherically symmetric nuclear field Hamiltonian H_{nucl} restores the spherical symmetry broken in a nuclear deformed Hartree state so that it gives rise to the nuclear collective tunneling transitions between deformed Hartree states: The residual interaction Hamiltonian H_{sd}, acting as the restoring force for the spherical symmetry of the nuclear system to be recovered, gives rise to the collective tunneling transitions between prolate Hartree states and oblate Hartree states. Since the spherical intermediate states are highly excited lying at the top of the energy surface of the nucleus ^{28}Si, the spherical symmetry broken in a prolate Hartree state is restored so that the nucleus tunnels to an oblate Hartree state, proceeding through tri-axially asymmetric deformed states. The ground state of the nuclear Hamiltonian H_{nucl} in Eq. (15) contains not only the Hartree states in Eq's. (44)-(49) but also the tri-axially asymmetric deformed states such as in Eq's. (50) and (54).

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